Lattices and Boolean Algebras

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Abstract

In this work , we study the main properties of lattices and Boolean Algebras ,we give various examples. In particular we study the finite Boolean Algebras . Indeed B it is isomorphic to the power set $P(X)$ for some x , where X is the power set of atoms in B .
Finally , as an application we study the algebra of electrical circuits .The set of all circuits ,form a Boolean Algebras .

Project Objectives:

1. To know the structure of lattices and Boolean Algebras .
2. To classify finite Boolean Algebras up to isomorphism .
3. Study the electrical circuit as an application of Boolean Algebras .

Results:

1. lattice and Boolean Algebras is an Algebraic structure of (B , $\wedge$ $\vee$,)
2. Finite Boolean Algebra is isomorphic to the power set $P(X)$.
3. The set of all circuits ,form a Boolean Algebras .

Analysis

1- properties and structure of poset Lattices and Boolean Algebra .

2- our project include many definitions :

•A lattice is a poset L such that every pair of elements in has L a least upper bound and a greatest lower bound.

• A Boolean algebra is a lattice B with a greatest element I and a smallest O element such that is both distributive and complemented.

• A Boolean algebra is a finite Boolean algebra if it contains a finite number of elements as a set.

3 – We use many theories :

* Let L be a nonempty set with two binary operations $\wedge$ and $\vee$ satisfying the commutative, associative, idempotent, and absorption laws.
We can define a partial order on L by a $\prec$ b iff a $\vee$ $b = .b$ Furthermore , L is a lattice with respect to $\prec$ if for all a,b $\in$ L , we defi-ne the least upper bound and greatest lower bound of a and b by a $\vee$ b , a $\wedge$ b and , respectively.

* Let B be a finite Boolean algebra. Then there exists a set X such that B is isomorphic to P(X) .

Example

Let X be any set .We will define the **power set** of X to be the set of all subset of X. We denote the power set of X by P(X) for example,
let X = {a ,b, c}.Then P(X) is the set of all subset of the set {a ,b,c} :
On any power set of a set , set inclusion $\subseteq$ is a partial order. We can represent the order on {a,b,c} schematically by a diagram such as the on in Figure 1.1

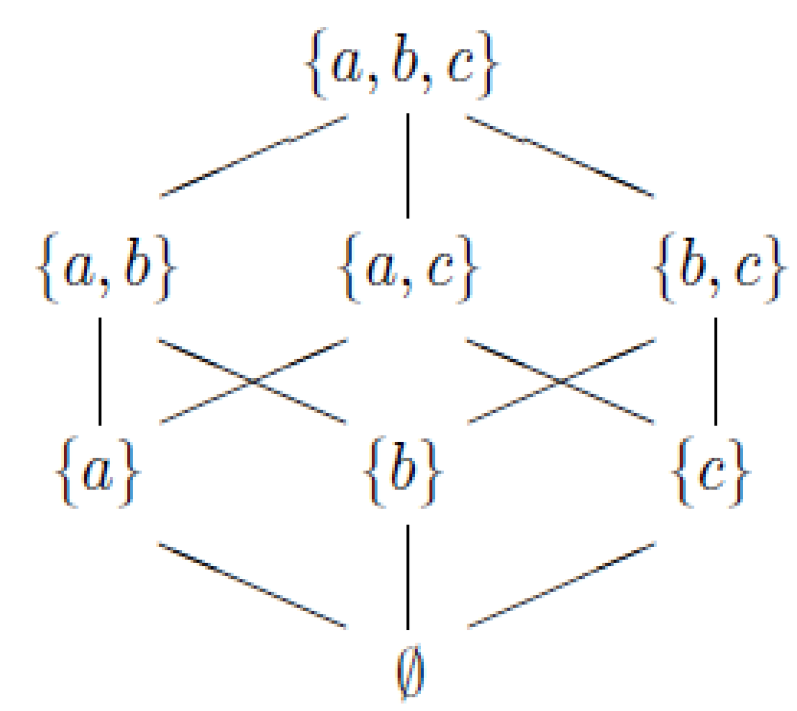

Figure 1.1 partial order on P ({a ,b,c}).

The graph shows the general outline of the steps of the project .